High-resolution seismic time-strain estimation

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Summary

This study introduces and evaluates two seismic time-strain inversion methods: total variation (TV), regularised inversion, and joint inversion with segmentation (JIS). Both methods effectively recover seismic time-strain while suppressing noise. Synthetic data experiments highlight the superior performance of JIS, which provides cleaner, more accurate estimates and segments the inverted strain into user-defined classes, enhancing interpretability and offering an indirect measure of uncertainty. Using a warm start, JIS achieves faster convergence and improved computational efficiency. These methods represent a robust framework for high-resolution, interpretable seismic analysis, with future work focusing on extending them to 3D field data for broader validation and scalability.

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Introduction

The Earth's subsurface is changing at a local scale due to human activities, such as natural resource extraction or gas injection for purposes like carbon dioxide (CO_2) sequestration. Identifying and quantifying such changes is critical for applications such as hazard assessment, reservoir management, and energy storage. Changes in the subsurface manifest as variations in seismic velocity and/or physical strain, which in turn produce time shifts between time-lapse seismic datasets. Therefore, quantifying these time shifts provides an indirect measurement of subsurface changes. However, time shift estimation is a non-linear inverse problem (Rickett et al., 2007; R. Williamson et al., 2007; Grandi et al., 2009) that is highly ill-posed and complicated by factors such as data noise and non-repeatable seismic acquisition conditions. Therefore, advanced regularisation or precondition techniques are needed for robust time shift estimates.

An alternative to directly inverting for time shifts is to invert for time strains (Edgar and Blanchard, 2015; Taweesintananon et al., 2024), which correspond to the rate of change of time shifts with respect to the vertical axis (e.g. two-way-time). This approach offers several advantages: (1) time strains reflect the fractional change in velocity and/or thickness of subsurface layers between seismic surveys, (2) time strains provide more localised and interpretable information about changes within the reservoir, (3) time strains are instrumental in quantifying reservoir compaction, overburden expansion, and variations in pore pressure and fluid saturation. Moreover, in the time strain domain, advanced regularisation techniques such as surface constraints (Edgar and Blanchard, 2015), can be applied more effectively. Similarly, Total Variation regularisation (TV) provides a robust mathematical framework that promotes piecewise smooth solutions while preserving critical discontinuities, enabling more accurate time strain reconstruction. In this study, we present a methodology to include TV regularisation by leveraging proximal operators. Furthermore, we propose to include another regularisation strategy known as joint inversion and segmentation (JIS), which has demonstrated exceptional results in both static and dynamic post-stack inversion. A unique by-product of JIS lies in its ability to simultaneously provide a segmentation of the strain, which serves as an indirect measure of uncertainty in the inverted strain. The effectiveness of these methodologies is validated on synthetic data, demonstrating their potential for improved resolution and interpretability in seismic analysis.

Theory and Methods

The relationship between spatially equivalent traces in the baseline $d_1(t)$ and monitor $d_2(t)$ seismic datasets, as described by Rickett et al. (2007), is given by:

$$d_1(t) \approx d_2(t + \tau(t)), \tag{1}$$

where τ(t) is the time shift field. This time shift equation assumes every trace in the post-stack seismic dataset as a near-zero offset. Time strain, defined as the rate of change of time shift along the vertical axis, is related to the time shift field as:

$$\tau_N = dt\, C\, u, \tag{2}$$

where C is an [N × N] causal integration operator. Incorporating the time strain field u into Equation (1) and including an amplitude correction term, as suggested by Williamson, P.R. et al. (2007), yields:

$$d_1(t) \approx d_2(t + dt\, C\, u) + G\, u, \tag{3}$$

where $G\, u$ represents a post-stack modeling operator that accounts for residual amplitude corrections. Estimating the time strain field u from Equation (3) constitutes a nonlinear optimisation problem. In this work, we solve this problem using Gauss-Newton iterations, which iteratively minimise the following functional:

$$\underset{\Delta u}{\operatorname{argmin}} \left\| d_1(t) - d_2(t + dt\, C\, u_{i-1}) - G u_{i-1} - \left(G + dt\, J_{d_2} C\right)\Delta u \right\|_2^2 + \lambda \|\Delta u + u_{i-1}\|_{TV}, \tag{4}$$

where J_{d_2} is the Jacobian of d_2 with respect to u. TV regularisation enforces sparsity in the gradients of the estimated time-strain field, promoting sharp and well-defined boundaries. The parameter λ serves as a regularisation weight, balancing the trade-off between data fidelity and the smoothness or sparsity imposed by the TV regularisation.

Joint Inversion with Segmentation (JIS)

The regularisation can be further enhanced to incorporate segmentation terms, allowing the inversion process to simultaneously classify the resultant time strains into user-defined classes. This approach has demonstrated state-of-the-art performance in seismic inversion problems (Ravasi and Birnie, 2021; Romero et al., 2022). The JIS framework is formulated as:

$$\underset{\Delta u}{\operatorname{argmin}} \|\mathrm{Op}\ \Delta u - d\ \|_2^2 + \alpha\|\Delta u + u_{i-1}\|_{TV} + \delta \sum_{j=1}^{N_c} \sum_{i=1}^{N_x N_z} V_{ji} \left((\Delta u + u_{i-1}) - c_j \right)^2 + \beta \sum_{j=1}^{N_c} \left\| V_j^T \right\|_{TV}, \tag{6}$$

Where:

$$\mathrm{Op} = -\left(G + dt\, J_{d_2} C \right) \quad \text{and} \quad d = -(d_1(t) - d_2(t\ + dt\ C\ u_{i-1}) - G u_{i-1})$$

In this formulation, the first two terms correspond to the TV-regularised inversion described in Equation (4), while the last two terms are designed to optimise the new variable V. The variable V is a segmentation matrix, where each column represents the membership probability of a model point belonging to a specific class, constrained to the unit simplex. The class vector $c = [c_1, c_2, \dots, c_{N_c}]$ is a user-defined vector specifying the time-strain magnitudes that the inverted time-strain will be segmented into. The fourth term in Equation (5) imposes TV regularisation on the V matrix, promoting smoother segmentation boundaries and favouring larger, contiguous regions over smaller, isolated partitions. Although Δu and V are nonlinearly related, the functional in Equation (5) becomes convex with respect to one variable when the other is held fixed. This property enables a splitting approach, where the optimisation alternates between solving for Δu and V in two separate convex optimisation steps. The Δu subproblem can be formulated as a standard optimisation problem of the form:

$$\underset{\mathrm{x}}{\operatorname{argmin}}\ f(Kx + b) + g(x)$$

where $f(x)$ is a non-smooth convex function, $g(x)$ is smooth and convex function, and K is a linear operator. To solve this type of problem, we employ the Primal-Dual algorithm (Chambolle and Pock, 2011), a specialised proximal solver that achieves an O(1/N) convergence rate in finite-dimensional spaces. While the original formulation of the Primal-Dual algorithm addresses problems of the form $f(Kx) + g(x)$, we extend it here to handle the modified functional $f(Kx + b) + g(x)$, leveraging the proximal properties of the algorithm to adapt it to this context. The full derivation of this approach will be detailed in an upcoming paper.

Results

In this study, we utilise the Hess 4D synthetic model, which features a time-lapse velocity change localised at the top of the right anticline structure. For detailed information on the model, refer to Romero et al. (2022). The synthetic baseline and monitor velocity models are then converted to time domain using a simple depth-to-time interpolation. The corresponding time-lapse seismic data is modelled using a Ricker wavelet with peak frequency of 8Hz, and band-limited Gaussian noise is independently added to both the baseline and monitor datasets to simulate realistic conditions. Figure 1a–c depicts the noisy baseline and monitor synthetic datasets alongside their difference image. The difference image highlights three key features: (1) amplitude variations corresponding to velocity changes at the top of the anticline, (2) time shifts affecting all reflectors beneath the region of velocity change, and (3) the presence of non-repeatable noise. Figure 1d shows the synthetic time strain caused by velocity changes between surveys, which serves as the ground truth and benchmark for evaluating inversion results.

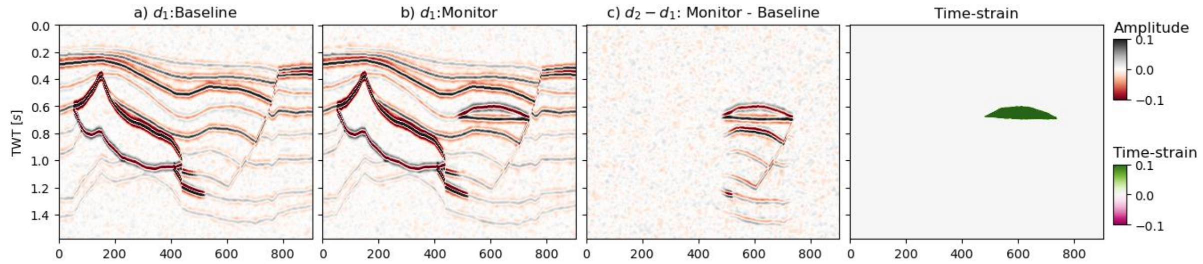

Figure 1 *(a) Baseline seismic data, (b) monitor seismic data, (c) time-lapse difference between the monitor and baseline datasets, and (d) synthetically modelled time strain.*

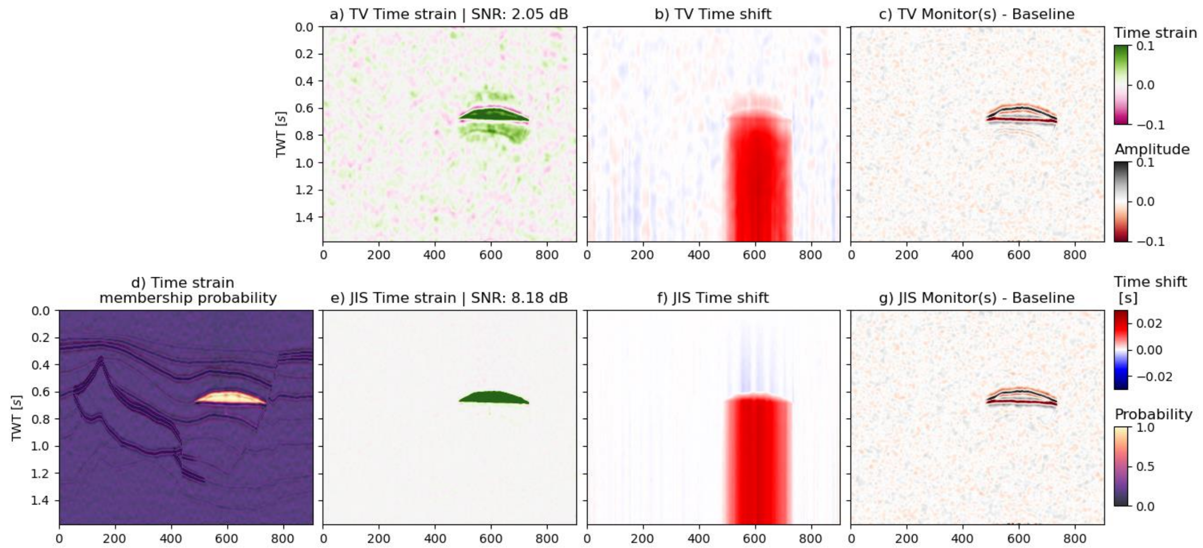

Figure 2 *(a) Time strain inverted using TV inversion, (b) time shift derived from the inverted time strain, (c) seismic time-lapse difference image after time shift correction of the monitor dataset using TV inversion, (d) time strain class +0.1 membership probability map from JIS, (e) time strain inverted using JIS, (f) time shift derived from the JIS-inverted time strain, and (g) seismic time-lapse difference image after time shift correction of the monitor dataset using JIS. The lateral axis represents the trace number which spacing is 10 m.*

Figures 2a–c present the results of the TV inversion for time strains, the computed time shifts derived from the inverted time strain, and the difference image after time shift correction. The TV inversion successfully captures the location and overall shape of the time strain; however, some side lobes—artifacts unrelated to the true strain—remain visible, and noise suppression is incomplete. Despite these limitations, the time shifts computed from the time strain effectively correct most of the shifted reflectors in the monitor survey, as evident from the improved difference image. Figures 2d–g demonstrate the performance of the JIS method. In this case we use three time strain classes: $[-0.1, 0, 0.1]$. In our implementation, we performed 10 iterations of TV inversion as an initialisation step before applying the JIS algorithm for 5 more iterations. This two-step approach yielded significant improvements: the time strain was recovered with near-perfect accuracy, achieving a very high signal-to-noise ratio (SNR), and non-repeatable noise was largely suppressed. The enhanced strain recovery facilitated a clean and smooth estimation of time shifts, which effectively corrected the shifted reflectors in the monitor dataset. Furthermore, the JIS algorithm provided a segmentation of the time strain field, offering an indirect measure of uncertainty in the classification of time strains as it assigned a membership probability for every point in the model to belong to each of the input classes. Figure 3 compares the convergence behaviour of three methods: TV inversion, JIS, and JIS with a warm start using 10 Gauss-Newton iterations of TV inversion (referred to as JIS-w). The results clearly demonstrate the benefit of initialising JIS with TV inversion, as JIS-w achieves faster and more stable convergence. All experiments were conducted on a machine equipped with an AMD EPYC 7713 64-Core Processor using the PyProximal library (Ravasi et al., 2024).

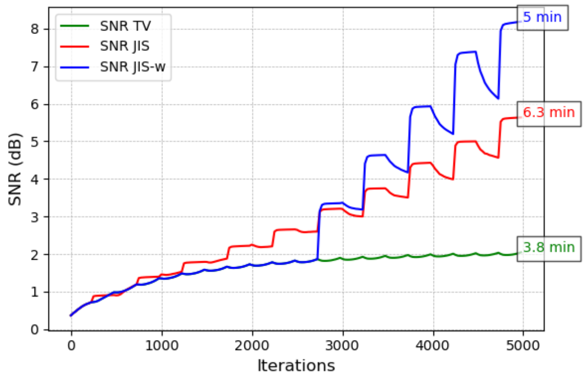

***Figure 3**: SNR evolution over iterations for TV inversion, JIS, and JIS with a warm start (JIS-w). The total computational time for each method is indicated alongside the respective curves.*

Conclusions

In this study, we introduced and evaluated two methods for seismic time-strain inversion: total variation (TV) regularised inversion, utilising proximal solvers, and joint inversion with segmentation (JIS). Both approaches demonstrated high accuracy in recovering seismic time-strain while achieving significant noise suppression. Synthetic data experiments revealed that the JIS method outperforms TV inversion, providing cleaner and more accurate estimates of seismic time-strain and time shifts. Additionally, JIS segments the strain into user-defined classes, offering enhanced interpretative value and an indirect measure of uncertainty. When paired with a warm start, JIS further improves convergence and computational efficiency. In conclusion, the JIS approach represents a robust and versatile tool for time-lapse seismic inversion, delivering high-resolution, interpretable results. Future work will apply these methods to 3D field data applications for broader validation and scalability.

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