Dynamic Fourier Ptychography via Space-Time Optimization

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Abstract: We introduce a dynamic Fourier ptychography (FP) technique via a spacetime framework, jointly recovering object states and deformation fields for detailed observation of rapid, complex behaviors in living cells.

1. Theory

Fourier ptychography (FP) is a highly effective microscopy imaging technique, ideal for the detailed reconstruction of live cells [1]. This study focuses on reconstructing the dynamic states of objects across successive frames. The complete pipeline is depicted in Fig. 1(a). Our method minimizes the error between the observed measurement $\mathbf{b}_t$ and the object state $\mathbf{o}_t$ after the forward imaging process $\mathbf{A}_t$ for each frame t. The ill-posed nature of this problem necessitates regularization to impose priors of the motion. We first impose a deformation prior by introducing motion fields between successive frames. The values of $\mathbf{o}_t(\mathbf{r})$ keep constant during its motion to $\mathbf{o}_{t+1}(\mathbf{r}+\mathbf{v}_t)$. We compute it by warping $\mathbf{o}_{t+1}$ with the flow $\mathbf{v}_t$, denoted as $\text{warp}(\mathbf{o}_{t+1}, \mathbf{v}_t)$. Next, to fit in situations where the object undergoes fast deformations, we incorporate the warp and projection method [2]. During each time t, we capture two images $\mathbf{b}_t, \mathbf{b}_{t+1/2}$ under random multiplexed illumination. Traditional strategies [3,4] follow the assumption of quasi-static object geometry, i.e. that the difference between $\mathbf{o}_t$ and $\mathbf{o}_{t+1/2}$ is negligible so that back projection of these two images adequately recovers the object. Our method utilizes the images differently. Given the estimated frame $\mathbf{o}_t$ and its motion field $\mathbf{v}_{t-1}$, we can approximate the $\mathbf{o}_{t-1/2}$ by warping the frame $\mathbf{o}_t$ backward in time. The forward warping can be calculated similarly. We introduce backward and forward warping operators as $\mathbf{W}_t^b$ and $\mathbf{W}_t^f$, and their processing can be expressed as $\mathbf{W}_t^b\mathbf{o}_t$ and $\mathbf{W}_t^f\mathbf{o}_t$. The joint optimization framework is:

$$\begin{aligned}(\hat{\mathbf{o}}, \hat{\mathbf{v}}) = \arg\min_{\mathbf{o},\mathbf{v}} \sum_{t=1}^{\mathrm{T}} \|\mathbf{A}_t\mathbf{o}_t - \mathbf{b}_t\|_2^2 + \alpha \sum_{t=1}^{\mathrm{T}-1} \|\text{warp}(\mathbf{o}_{t+1}, \mathbf{v}_t) - \mathbf{o}_t\|_1 \\ + \gamma\left(\sum_{t=1}^{\mathrm{T}-1}\left\|\mathbf{A}_{t+}\mathbf{W}_t^f\mathbf{o}_t - \mathbf{b}_{t+}\right\|_2^2 + \sum_{t=2}^{\mathrm{T}}\left\|\mathbf{A}_{t-}\mathbf{W}_t^b\mathbf{o}_t - \mathbf{b}_{t-}\right\|_2^2\right) + \mathscr{L}_{\mathbf{v}} + \mathscr{L}_{\mathbf{s}},\end{aligned} \tag{1}$$

where the first term is the data fidelity term, the second term is optical flow constrain, the third integrates the warp and projection method, the fourth is for velocity field regularization, and the final one involves applying the Huber norm on both spatial and temporal derivatives. We simplify $\mathbf{b}_{t+1/2}$ and $\mathbf{b}_{t-1/2}$ as $\mathbf{b}_{t+}$ and $\mathbf{b}_{t-}$. $\mathbf{A}_{t+}$ and $\mathbf{A}_{t-}$ represent the corresponding forward imaging functions. α and γ are weights of the different terms.

We address the joint optimization problem in an alternating mode between the object set $\mathbf{o}$ and the flows $\mathbf{v}$ of all the frame times. The deformation field estimation task can be expressed as:

$$\hat{\mathbf{v}} = \arg\min_{\mathbf{v}} \alpha \sum_{t=1}^{\mathrm{T}-1} \|\text{warp}(\mathbf{o}_{t+1}, \mathbf{v}_t) - \mathbf{o}_t\|_1 + \mathscr{L}_{\mathbf{v}}. \tag{2}$$

The v-update solver utilizes a multi-scale strategy [5,6] to handle large displacements. On the other hand, the sub-problem for the object reconstruction is to minimize the complex objective function that balances data fidelity and all the other regularization terms except $\mathscr{L}_{\mathbf{v}}$ in Eq. (1). The ADMM framework [7] is employed for solving this problem.

2. Experimental Demonstration

We conduct an FP setup following the scheme in Fig. 1(b). The LED matrix (Adafruit, 4mm pitch) is placed at ~75 mm away from the specimen plane. The central wavelength of the green channel is 525 nm. We use a microscope objective (Mitutoyo, M Plan Apo 10x, 0.28 NA) and a tube lens (Thorlabs, TTL200-B) to form the image at the sensor (Triton, 5.4 MP Sony IMX490 CMOS, 3 µm pixel size).

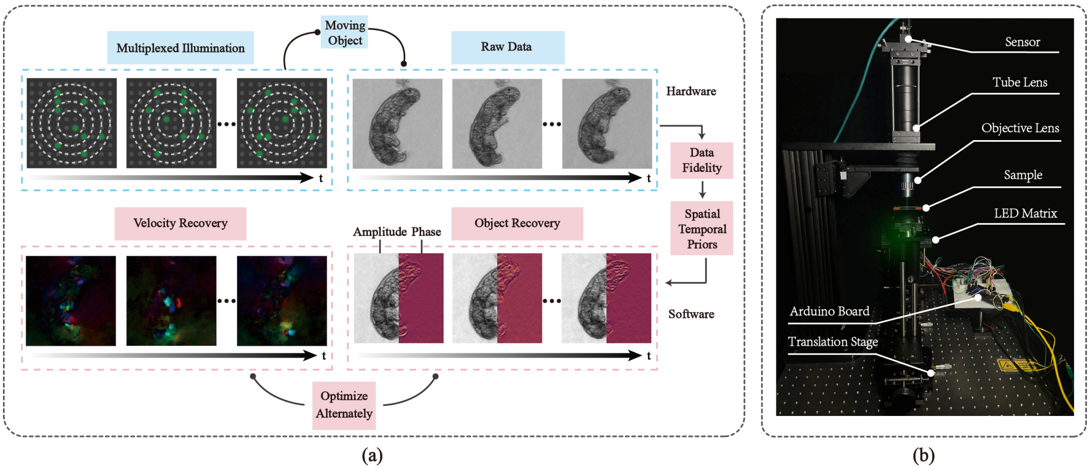

Fig. 1: Principle of dynamic FP via spacetime framework. (a) Overall pipeline. (b) Experimental setup.

Figure 2 presents a comparative analysis of the reconstruction of a live rotifer. The traditional way employs data from multiple times to reconstruct a single frame, leading to significant artifacts from the time-mixed information. In contrast, our method utilizes a single-frame time image for reconstruction. The spatial and temporal regularization in the optimization process demonstrates satisfactory performance in both amplitude and phase results.

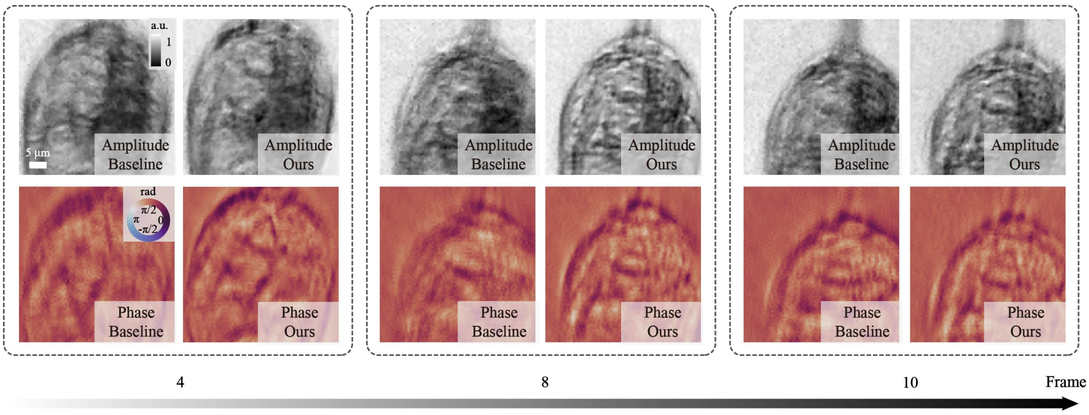

Fig. 2: Results comparison of a live rotifer at different times.

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